Bias Correction for characterizing transparent Particles using the Time Shift Technique

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Abstract

Characterizing droplets in spray processes is of high interest in many areas, such as car painting or spray drying. The Time Shift technique provides an efficient and accurate way to measure size and velocity of individual droplets in sprays. Usually peak detection algorithms are applied to extract the information of interest out of the acquired signals. In this work we show that peak detection algorithms are biased when estimating size and velocity of transparent droplets measured by the Time Shift technique. The bias magnitude is quantified with respect to different droplet sizes and velocities as well as aperture configurations. Additionally, based on interpolation, a practical method is provided to reduce the bias. Using the proposed approach, we show that the accuracy in sizing droplets is highly increased, especially for small droplet sizes.

Keywords: Spray, atomization, measurement, drop size, time shift, peak detection, bias correction

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Introduction

Spray characterization is important for a variety of applications, such as spray painting or spray drying, whereby the efficiency and quality of the spray process depends on atomization parameters like flow rate, injection pressure, airflow rate etc. that directly influence drop sizes and velocities. For instance, in coating processes, small droplets lead to overspray, whereas large drops lead to surface defects. Therefore spray characterization methods are essential tools for quality assurance, development and optimization of these processes; a review of measurement methods and corresponding techniques for spray characterization is available in Tropea (2011) [10].

The time shift technique (TS) provides an efficient and accurate method to measure size and velocity of individual droplets in sprays [4,5,6,7]. It was first introduced by Semidetnov (1985) [9] and was further developed by Damaschke *et al*. (2002) [1] und [3]. This technique is based on the light scattering of drops passing a shaped laser beam. The scattered light is detected by two sensors located at different scattering angles. The time shift between two acquired signals directly depends on drop size and velocity. Usually, peak detection algorithms are applied to measure this time shift.

However, up to now, processing of small transparent droplets has not been studied in detail. In the present work we show that applying peak detection algorithms (PD) for measuring drop sizes and velocities using the time shift technique leads to a biased estimation, especially for small drops. This is due to the fact that the acquired refractive orders of light, begin to overlap for small drops and lead to a systematic under-estimation of the drop size and velocity. The magnitude of this bias is quantified with respect to different droplet sizes and velocities as well as for different optical aperture configurations. We demonstrate that the error in velocity estimation is negligible, while there is a strong impact on drop sizes. A practical method is introduced to reduce the bias in real time. Besides a highly increased accuracy in drop sizing, this novel approach allows considerable flexibility in aperture configurations. For instance larger working distances are available while minimizing the measurement error.

Measurement Principle

We will briefly summarize the principles that are relevant for further discussions; a detailed description of the time-shift technique can be found in Albrecht et al. (2003) [1]. The time shift technique is based on light scattering from drops passing through a shaped laser beam. The aim is to determine size and diameter of the drops. **Figure 1** shows the sensor configuration, where the region of interest is defined by two and polarized Gaussian laser beams (Light Source 1 and Light Source 2 and by four detectors denoted by A, B, C and D.

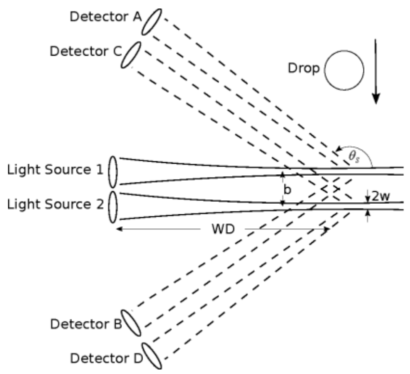

Figure 1 Sensor setup with working distance WD, distance between two laser beams b, scattering angle Θ_S and beam width $2w$.

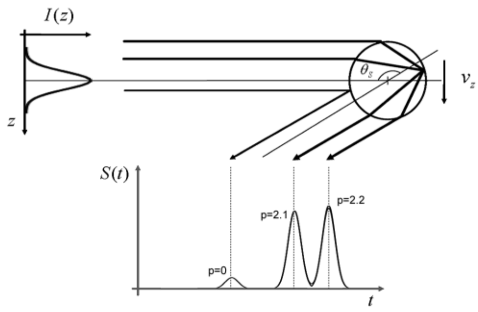

Figure 2 Time signal, generated by a spherical water drop with velocity v_z

The two laser beams are parallel and perpendicular polarized, respectively, where detectors A and B capture the perpendicular polarized light while detectors C and D capture the parallel polarized light. Whenever a drop passes a laser beam, it transforms the intensity of the laser beam in space into a time dependent signal on one of the detectors. Each detector measures a time dependent signal that is composed of the sum of all scattering orders p of the incident rays. **Figure 2** depicts an example measurement signal generated by a spherical water drop for a sensor configuration placed near the primary rainbow angle, where two modes of second-order refraction can be expected. It is mainly composed of a reflection ($p = 0$) and second order reflection with their respective modes ($p = 21$ and $p = 2.2$). For a Gaussian intensity profile $I(z)$ of the focused input beams the associated time shift signal $s(t)$ at a single detector at time t can be written as

$$s(t) = \sum_{\substack{p=0\\p=2.1\\p=2.2}} A_p(m, \Theta_S) \exp\left(-\frac{2\left(t - t_0^{(p)}\right)^2}{(w/v)^2}\right), \tag{1}$$

where w is half the laser beam width, $t_0^{(p)}$ the time instant when the scattering order p results in a maximum amplitude on the detector, v the drop velocity and $A_{\mathrm{p}}(m, \Theta_S)$ the intensity of scattering order p with relative refractive index m and scattering angle Θ_S. The intensities $A_p(m, \Theta_{\mathrm{S}})$ of the individual scattering orders are given in [4]. Notice that each detector measures the individual scattering orders at different time instances, depending on velocity and size of the drop. Hence, among others, the time differences between different sensor pairs are used to determine both parameters of interest (**Figure 3**). Note that for convenience Detector 1 stands for either detector A or C while Detector 2 represents detector B or D.

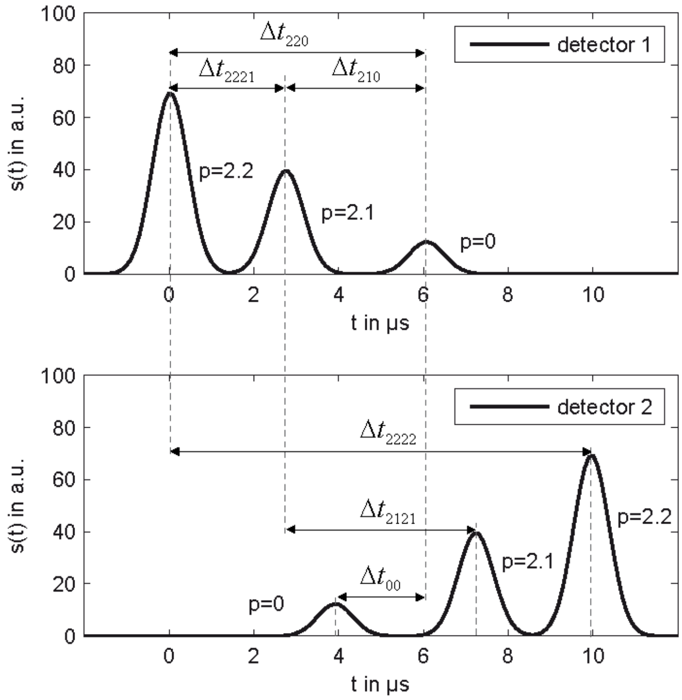

Figure 3 Time shift signals at Detector 1 and Detector 2

The time differences are given by

$$\Delta t_{00} = \frac{d}{v}\cos(\Theta_S/2) \tag{2}$$
$$\Delta t_{2121} = \frac{d}{v}\sin\left(\Theta_i^{(p=2.1)}(\Theta_S, m)\right) \tag{3}$$
$$\Delta t_{2222} = \frac{d}{v}\sin\left(\Theta_i^{(p=2.2)}(\Theta_S, m)\right) \tag{4}$$
$$\Delta t_{TOF} = \frac{l}{v}, \tag{5}$$

where l is the distance between the two laser beams, d the drop size and $\sin(\Theta_i)$ indicates the relative position between incident points on the surface of the particle [8]. Using geometric optics, $\sin(\Theta_i)$ is evaluated numerically by solving

$$\sin(\Theta_i^{(p)}) = m\sin\left(\frac{\pi}{2p} - \frac{\Theta_S}{2p} + \frac{\Theta_i^{(p)}}{p}\right) \qquad p \in [2, 4, 6, \ldots] \tag{6}$$
$$\sin\left(\Theta_i^{(p=0)}\right) = \cos\left(\frac{\Theta_S}{2}\right). \tag{5}$$

Error Description

A typical way to determine size and velocity of a droplet is to estimate the maximum peak positions t_0 for each detector, compute the time differences and finally apply Equations (2-5).

For water drops with $m = 1.33$ and scattering angle $\Theta_S = 165°$, the peak amplitudes for each drop are $A_{(\mathrm{p=2.1})} > A_{(\mathrm{p=2.2})} > A_{(p=0)}$. Thus, in practice, the peak position $t_0^{(p=2.1)}$ for each detector is used to determine the time differences, i.e.

$$\Delta t_{2121,\mathrm{AC}} = t_{0,\mathrm{A}}^{(p=2.1)} - t_{0,\mathrm{C}}^{(p=2.1)} \tag{7}$$
$$\Delta t_{2121,\mathrm{BD}} = t_{0,\mathrm{B}}^{(p=2.1)} - t_{0,\mathrm{D}}^{(p=2.1)} \tag{8}$$
$$\Delta t_{\mathrm{TOF,AB}} = t_{0,\mathrm{B}}^{(p=2.1)} - t_{0,\mathrm{A}}^{(p=2.1)} \tag{9}$$
$$\Delta t_{\mathrm{TOF,CD}} = t_{0,\mathrm{D}}^{(p=2.1)} - t_{0,\mathrm{C}}^{(p=2.1)}, \tag{10}$$

where the subscripts A, B, C, D have been used to denote the relation to the respective detectors A, B, C, D. Typically peak detection algorithms are applied to determine the maximum peak position $t_{\max}$ that corresponds to $t_0^{(p=2.1)}$. Note that using four different detectors result in redundancy when estimating the time shifts. Thus, Δt_{2121} and Δt_{TOF} are usually computed by averaging over the obtained quantities from Equations (2-5).
Now it is clear that an overlap of scattering orders leads to a smeared measurement signal, where each individual scattering order may not be accurately resolved (**Figure 4**).

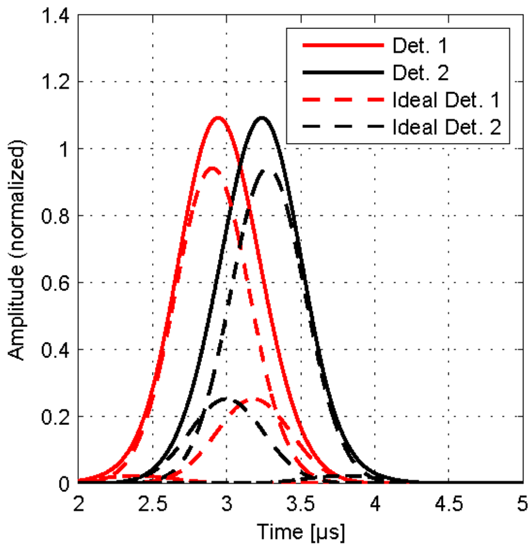

Figure 4 Smeared measurement signals at detector 1 (Det. 1) and detector 2 (Det. 2) composed of their individual scattering orders (ideal Det. 1 and ideal Det. 2).

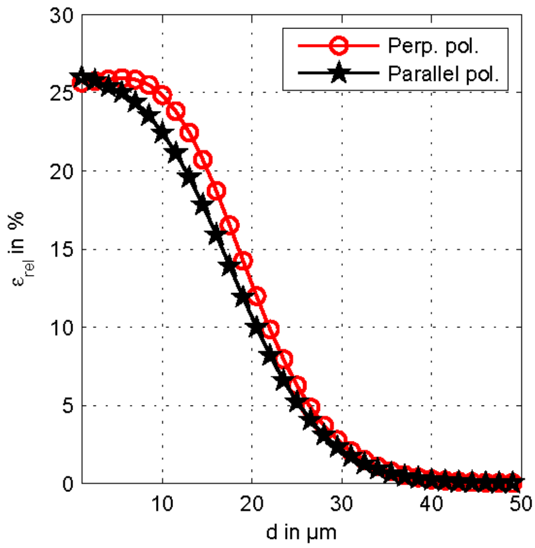

Figure 5 Relative error ε_{rel} for different drop sizes d for parallel polarized (Parallel pol.) and perpendicular polarized (Perp. pol.) laser caused by overlapping scattering orders.

However this systematic error is relatively small as long as the scattering orders do not overlap. This is one reason why the laser beam width and measurement range have to be chosen with care [8]. For instance most systems operate with a strongly focused laser beam width to resolve the captured scattering orders properly. Unfortunately one has to accept a small working distance since the beam width w and the focal length f are related by [5]

$$w = \frac{f\lambda}{2\pi w_0}, \tag{11}$$

where w_0 is the (fixed) initial radius of the Gaussian beam and λ the laser wavelength.

To analyze the impact of overlapping scattering orders on characterizing drops in detail we define a Peak Detection algorithm, or estimator $\hat{T}$ to identify or estimate the value $t_{\max}$. While we do not discuss different PD algorithms here, we assume that an unbiased estimator $\hat{T}$ for $t_{\max}$ is given [2], i.e. $E[\hat{T}] = t_{\max}$, where $E[]$ is the expected value.

For a given d and v, $t_{\max}$ is a solution of

$$\left.\frac{\partial s(t)}{\partial t}\right|_{t=t_{\max}} = 0, \tag{12}$$

where $t_{\max}$ needs to be obtained numerically. However it is easy to show that $t_0^{(p=2.1)}$ is in general not a solution of Equation (12), i.e. $\left.\frac{\partial s(t)}{\partial t}\right|_{t=t_0^{(p=2.1)}} \neq 0$. Since $E(\hat{T}) = t_{max}$ by assumption, $E(\hat{T}) - t_0^{(p=2.1)} \neq 0$, meaning the estimator is biased in identifying $t_0^{(p=2.1)}$. Note that this systematic error is dependent on various parameters such as drop size, drop velocity, beam width, scattering angle, etc. As we are interested in characterizing drops rather than in the exact position of a particular scattering order, we focus on the impact that the bias has on the measured drop size d_{m} and measured velocity v_{m}. Apart from different polarizations, detector A and B measure a signal which is similar in structure. Hence the systematic error in velocity estimation is negligible, since both

$t_{\mathrm{max,A}}$ and $t_{\mathrm{max,B}}$ are shifted from the corresponding refractive order in the same direction. In contrast, the impact on drop sizing is crucial since the mirrored signal measured by detector A is received at detector C, such that $t_{\mathrm{max,C}}$ is over estimated while $t_{\mathrm{max,A}}$ is underestimated (**Figure 4**). In the following the systematic error in drop sizing is denoted by ε, i.e.

$$\varepsilon = d_{\mathrm{m}} - d \qquad \text{with} \qquad d_{\mathrm{m}} = \frac{(t_{\mathrm{max,A}} - t_{\mathrm{max,C}})v}{\sin\left(\Theta_i^{(p=2.1)}(\Theta_S, m)\right)}. \tag{13}$$

The applied sensor uses parallel and perpendicular polarized light beams having a width of $\frac{w}{2} = 10\mu m$, respectively. The resulting working distance is $12cm$ and the detector angle Θ_S is 165°. **Figure 5** shows the relative error, i.e. $\varepsilon_{\mathrm{rel}} = \varepsilon/d$, for different drop sizes and for both light polarizations. We observe that especially for $d < 25\mu m$, the error in sizing drops goes up to 25% as the scattering orders begin to overlap. Drop sizes above $25\mu m$ are estimated with $\varepsilon_{\mathrm{rel}} < 5\%$ since the scattering orders are accurately resolved. One way to accurately size small drops is to reduce the beam widths of both lasers. However, the working distance will suffer (Equation 11) which is inacceptable for most practical applications.

Bias Reduction

In the following we present an efficient approach to reduce the systematic error. Having derived the systematic error in Equation (13), at least numerically, allows the measurements to be corrected. To keep computational costs feasible, Equation (13) is not solved for each measured drop individually. **Figure 6** shows the measured drop sizes d_{m} over the actual drop size d for the applied sensor system.

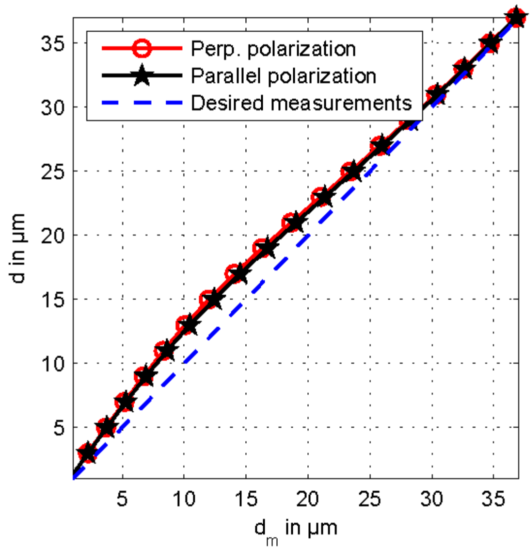

Figure 6 Measured drop sizes d_{m} over the actual drop size d for both polarizations.

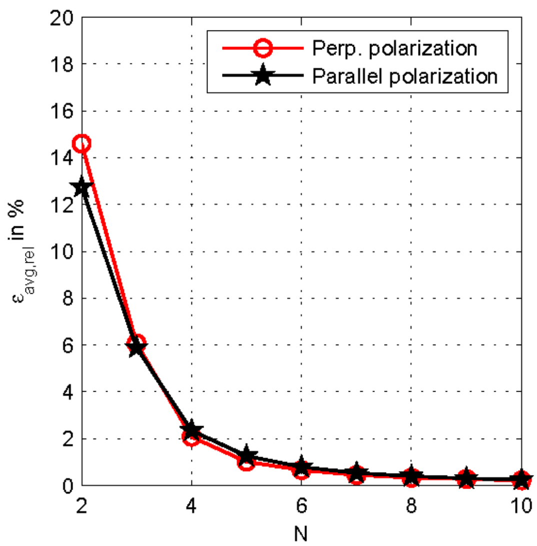

Figure 7 Average measurement error $\varepsilon_{\mathrm{avg,rel}}$ for different numbers of supporting points N.

The dashed line shows the (unbiased) desired measurement result, where each measured drop size d_{m} is equal to its correct value. The solid lines show the actual obtained measurements, being different from the dashed line due to the overlapping scattering orders.

The key idea is to interpolate the measurement curve in **Figure 6** by a fixed number N of points to find a function f s.t. $f(d_{\mathrm{m}}) \approx d$ for $d_{\mathrm{m}} \in [0, D]$, where D defines the maximum drop diameter that we want to correct. Whenever a new drop size is measured, its actual size can be corrected, which is explained in the following.

A priori a maximum measurement error $\varepsilon_{\max}$ is prescribed, which delineates an acceptable from an inacceptable measurement result in the domain for f. In other words, all measurements within the acceptable region are not corrected, since the error is below $\varepsilon_{\max}$.

Within the interval $[0, D]$, a lookup table with N points $(d_{\mathrm{m,k}}, d_k)$ is generated to build f, where $N = 2, 3, 4, \ldots$ and $k = 1, 2, \ldots, N$. In the next step the interval $[0, D]$ is divided into $N - 1$ equidistant parts by N points, where $d_{\mathrm{m,k}} = \frac{D}{N-1}(k-1)$ for $k = 1, \ldots, N$. The points $(d_{\mathrm{m,k}}, d_{\mathrm{k}})$ are computed in advance by using Equation (12) and

(13). Having stored all points in a lookup table, a new measured drop size d_{m} can be corrected by a linear interpolation. In particular, we use the obtained function for a given N

$$f_k^{(N)} = a_{\mathrm{k}} d_{\mathrm{m}} + b_{\mathrm{k}}, \tag{14}$$

where we chose k s.t. $d_{\mathrm{m,k}} < d_{\mathrm{m}} < d_{\mathrm{m,k+1}}$. The parameters a_k and b_k are now given by

$$a_{\mathrm{k}} = \frac{d_k - d_{\mathrm{k+1}}}{d_{\mathrm{m,k}} - d_{\mathrm{m,k+1}}} \tag{15}$$

$$b_{\mathrm{k}} = d_{\mathrm{k}} - d_{\mathrm{m,k}} a_k \tag{16}$$

for $k = 1, \dots, N$.

It is clear that an increasing N will lead to more accurate results while the computational complexity increases. Therefore we suggest to choose N, such that a maximum relative average error within $[0, D]$ is not exceeded, where we define the average error to be $\varepsilon_{\mathrm{avg,rel}}(N) = 1/D \int_{[0,D]} \frac{1}{d} \left(f_k^{(N)}(d_{\mathrm{m}}) - d \right) \delta d$. **Figure 7** shows the error decreasing for an increasing number of supporting points N. Now N is selected such that a maximum relative average error is not exceeded.

Results

In the following section we will apply the developed method in a simulation environment. We are working with a sensor system as in **Figure 1**, where $\Theta_S = 165°$, the distance $d = 10$μm and the two Gaussian laser beams are perpendicular and parallel polarized. Suppose we are measuring drop sizes in a water spray containing drops between $1\mu m$ and $100\mu m$. Again we focus on the measurement error $\varepsilon_{\mathrm{rel}}$ in sizing drops as defined before. Note that the given beam width results in a working distance WD which is $10cm$. To overcome the problem of overlapping peaks we proceed as described before:

- Define an in-acceptable region D that exceeds a maximum measurement error: $\varepsilon_{\mathrm{rel}} > 0.5\%$ which is the case for $1\mu m < d < 37\mu m$ (**Figure 5**)
- Define an average error that should not be exceeded: $\varepsilon_{\mathrm{avg,rel}} < 0.5\%$ which is the case for $N \geq 7$ (**Figure 7**)
- Set up the lookup table using Equation (12) and (13)
- Correct each measured drop using Equation (14)

Figure 8 shows the relative error in drops for different drop diameters with and without the performed bias reduction at a working distance $WD = 10cm$. We observe that the measurement error over the entire diameter range is decreased, dependent on the criteria that have been chosen. Additionally we increased the working distance from $10cm$ to $20cm$ while using similar criteria for bias reduction as before. Without bias reduction a working distance of $20cm$ is inacceptable since the measurement errors are strongly spread over a large measurement range (black curve). However, applying the presented bias reduction makes larger working distances much more attractive, since the measurement error is reduced over the entire measurement range.

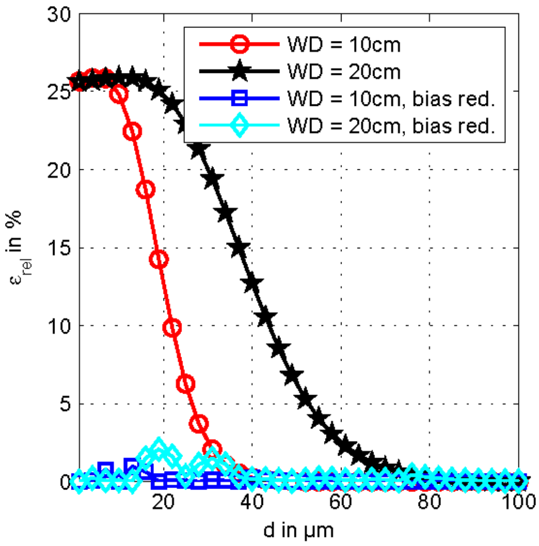

Figure 8 Relative error in sizing drops for different drop diameters and for different working distances (WD) with and without the presented bias reduction.

Summary and Conclusions

The present work describes the time shift technique for transparent particles, e.g. water drops. We showed that applying peak detection algorithms for measuring drop sizes and velocities using the time shift technique leads to a biased estimator. The reason is that the acquired refractive orders of light, especially for small droplets, overlap and lead to a systematic under-estimation of the drop size and velocity. We analyzed the error with respect to different droplet sizes and velocities as well as for different optical aperture configurations. Based on interpolation, a practical method is introduced to reduce the bias. We showed in simulations that besides a highly increased accuracy in drop sizing, this novel feature allows considerable flexibility in aperture configurations. For example the working distance can be increased while maintaining a small measurement error.

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