

The Time-Shift Technique for Measurement of Size and Velocity of Particles

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Abstract

A novel implementation of the time-shift technique for particle/droplet sizing is introduced which discriminates the polarization state of scattered light to determine refractive index and uses the signal width to estimate velocity. The particle size is determined directly from the time-shift between signals arising from the reflection and second-order refraction scattering orders. The system presented here utilizes a laser diode as a light source and is configured to operate in near backscatter; hence, a compact, relatively inexpensive and robust system for particle size, refractive index and velocity is presented.

Introduction

Particle, and in particular droplet sizing is of prime importance when characterizing sprays and other atomization processes. A multitude of techniques exist for determining size, including phase Doppler, ILIDS/IPI, Mie/LIF and techniques based on femtosecond illuminating pulses. A review of available techniques is available in Tropea (2011) [1]. Velocity is usually determined by combining the sizing technique with a laser Doppler or time-of-flight technique [2]. Very few techniques are available for determining refractive index of individual droplets *in-situ*, for example rainbow refractometry. Furthermore, most existing techniques for size measurements require *a priori* knowledge about the refractive index. Therefore, a need exists for a technique measuring size, velocity and refractive index in a single instrument, preferably for individual droplets and preferably in a near-backscatter configuration, where perhaps only one optical access to the measurement position is required. This was exactly the goal of the present work.

Measurement principle

The time-shift technique is used here in a somewhat modified form. This technique was first introduced by Semidetnov (1985) [3] and is further developed in Damaschke et al (2002) and Albrecht et al (2003). It is realized by creating an illuminated volume considerably smaller than the size of the particle to be measured, usually taking a Gaussian beam shape in intensity. Under these conditions the scattering from a spherical droplet can be interpreted according to various scattering orders by employing a Debye series [4] expansion of the Mie [5] scattering functions or using a geometric optics [6] approach to the scattering field. Effectively a different measurement volume is realized for each scattering order and each measurement volume is displaced in space from one another (Fig. 1, upper right). The magnitude of this displacement depends on the particle size, velocity, refractive index and the scattering angle. When the particle passes through these different measurement volumes, the displacement appears as a time shift of the signal on any one single detector.

The experimental setup presented here consists of two photo-detectors placed on one side of the laser beam at a scattering angle about 160deg (Fig.1, left). The collimated laser beam is focussed by two cylindrical lenses to a line with thickness approx. 30 μ m and width approx. 1000 μ m. The light scattered by the droplets/particles is split by a polarizing beam splitter into s- and p- polarized light and detected by the two photo-detectors separately. The signals coming simultaneously from two detectors provide (Fig. 1, lower right) the necessary information to estimate the size, velocity and refractive index of the droplets/particles.

The thickness of the laser beam should be smaller than the droplet size in order to obtain the characteristic time-shift signal (Fig. 2). If the laser line thickness is in the range or larger than the droplet/particle size, the time-shift signal is not suitable; the signal portions from different scattering orders overlap too much to be uniquely distinguished. The signal for a scattering angle of 160deg coming from one droplet is composed of several peaks, corresponding to scattering by the surface wave long path (SWLP), reflection ($p=0$) and two modes of second-order refraction ($p=2.1$, $p=2.2$). The interference between scattering orders does not occur, because they are temporally and spatially separated.

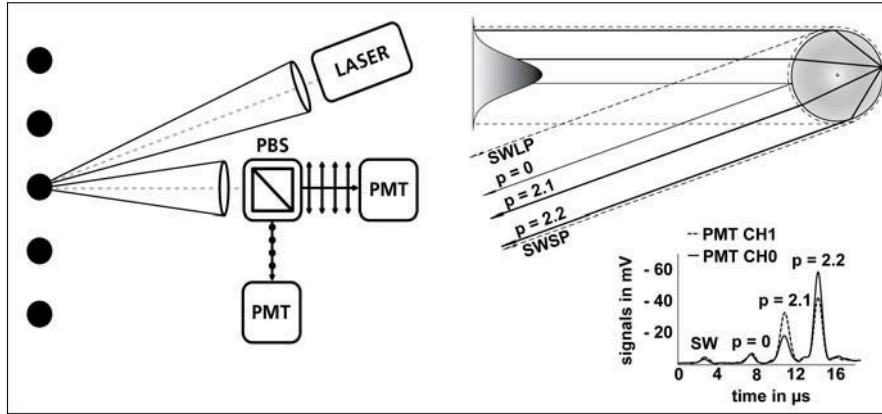


Figure 1: (left) Schematic illustration of the experimental setup; (upper right) Basic principle of time-shift technique; (lower right) Characteristic Time-Shift signal for different polarizations.

The occurrence time of the detected peaks (Fig.1, lower right) at a constant scattering angle depends on refractive index, size and velocity of the droplets/particles. The amplitude of the peaks in a time shift signal is given by the size of droplets. The amplitude will increase with increasing droplet size. The shape of each individual signal (surface wave, reflection, second-order refraction) takes the shape (and width) of the illuminating beam intensity profile and therefore depends only on the velocity of the droplets. The information, which can be gained from the time shift signal, is summarized in Table 1.

Table 1: Summary of information contained in the time-shift signal.

Signal feature	available information
Amplitude	d (size)
FWHM	v (velocity component normal to light sheet)
Polarization ratio	m (relative refractive index)
Time shifts	d, v, m

From geometrical optics, which is particularly applicable for larger droplets, it follows that the scattering intensities of all scattering orders are proportional to the square of the drop size at a constant refractive index and scattering angle, i.e.

$$I^{(p=0/p=2.1/p=2.2)}(d, \theta_s = \text{const}, m = \text{const}) \propto d^2 \quad (1)$$

Because the intensity of scattering orders is proportional to the square of the droplet size, the polarization ratio (or state of polarization (SOP)) of time-shift signals is only dependent on the refractive index. The polarization ratio can be written as

$$SOP = \frac{R_1^{p=0} + R_1^{p=2.1} (1 - R_1^{p=2.1})^2 + R_1^{p=2.2} (1 - R_1^{p=2.2})^2}{R_2^{p=0} + R_2^{p=2.1} (1 - R_2^{p=2.1})^2 + R_2^{p=2.2} (1 - R_2^{p=2.2})^2} \quad (2)$$

where R is a function of refractive index and scattering angle and the subscripts (1) and (2) indicate the polarization state and (p=0), (p=2.1), (p=2.2) the scattering orders.

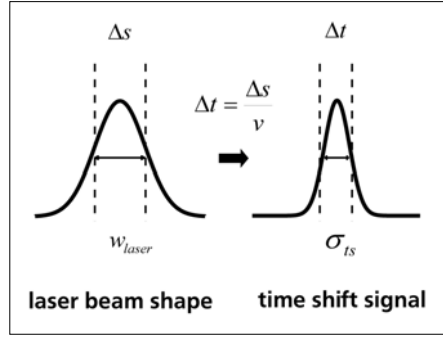


Figure 2: Laser beam in space and signal in time.

In order to estimate the velocity from the time-shift signal, the laser line thickness should be known and the FWHM (Full width half maximum) of the signals should be calculated. The width of the signal peak in time is directly related to the width of the illuminating beam in space (Fig. 2) so that the velocity of the particle can be computed according to

$$v = \frac{W_{Laser}}{\sigma_{ts}} \quad (3)$$

Afterwards the time shifts between peaks can be used to validate the estimated value of droplet size, refractive index and velocity. The time shift is given by

$$\Delta t_{01/02} = \frac{d/2}{v} \left(\cos\left(\frac{\theta_s}{2}\right) + \sin\left(\theta_i^{p=2.1/p=2.2}(\theta_s, m)\right) \right) \quad (4)$$

where the subscript (01) refers to the distance between reflection and first mode of second-order refraction and (02) the distance between the reflection and the second mode of second-order refraction (Fig. 3).

Signal validation for spray characterization

In general a spray is comprised of particles with a velocity and size distribution. For spray characterization the time-shift technique can only be applied if the detected time-shift signal originates from a single particle with a nearly spherical shape. For high density sprays, the measurement volume, which is limited by the laser line width, could illuminate more than one particle at the same time. This results in overlapping signals, which lead to errors in the analysis, as do signals coming from strongly non-spherical particles. Therefore a validation of the signal processing is required before calculation of particle size, velocity and refractive index. In this study a condition for signal validation is presented, which is termed the ‘gamma’ condition and which is based on a ray-tracing calculation.

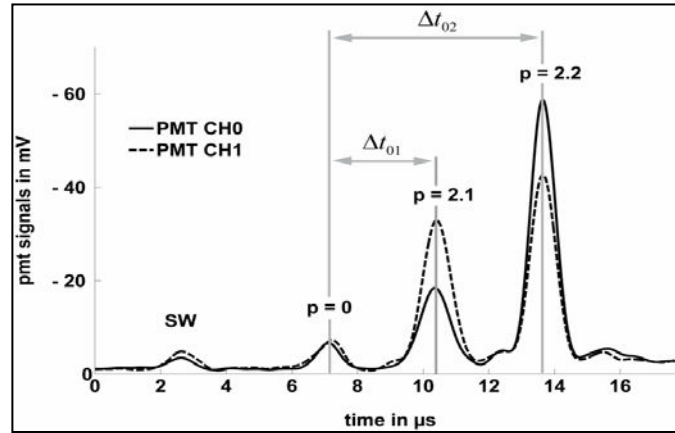


Figure 3: Typical time-shift signal.

Gamma is defined as a ratio between time interval between signal peaks of reflection and two modes of second-order refraction. These intervals can be calculated by employing geometrical optics and taking into account the velocity, size and refractive index of the particle and the scattering angle. This results the following relation for gamma:

$$\frac{\Delta t_{02}}{\Delta t_{01}} = \frac{\frac{d/2}{v} \left(\cos\left(\frac{\theta_s}{2}\right) + \sin\left(\theta_i^{p=2.2}(\theta_s, m)\right) \right)}{\frac{d/2}{v} \left(\cos\left(\frac{\theta_s}{2}\right) + \sin\left(\theta_i^{p=2.1}(\theta_s, m)\right) \right)} = \frac{\left(\cos\left(\frac{\theta_s}{2}\right) + \sin\left(\theta_i^{p=2.2}(\theta_s, m)\right) \right)}{\left(\cos\left(\frac{\theta_s}{2}\right) + \sin\left(\theta_i^{p=2.1}(\theta_s, m)\right) \right)} := \gamma(\theta_s, m) \quad (5)$$

Consequently gamma is constant for a constant refractive index and a fixed scattering angle. The numerical consideration (Fig. 4) provides values of gamma as a function of scattering angle for different relative refractive indexes. This allows validating signals for further signal processing and so reducing errors in the calculation of size and velocity of the droplets in a spray.

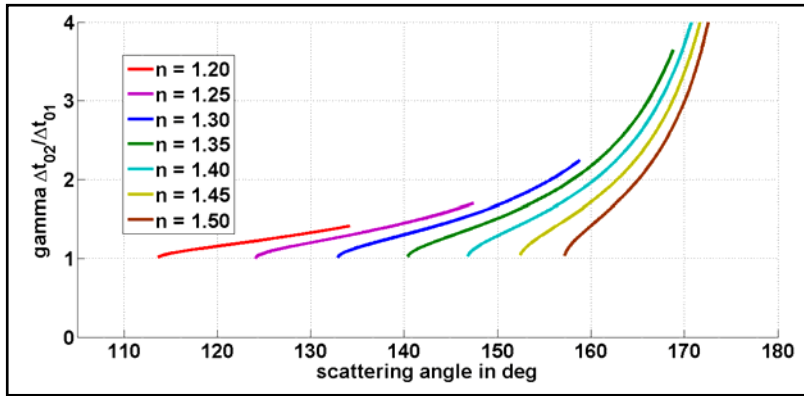


Figure 4: Numerical calculation of Gamma function

Experimental results

The measurements presented in this work were performed for spherical water droplets created by a mono-dispersed droplet generator. The droplet sizes were varied by changing pinholes with different diameters. For determination of the exact size of droplets and the space between them a strobed visualization was used. The velocity was calculated from time intervals between signals or from the frequency of the droplet generator and the space between droplets. The mean value of the velocity was around 10m/s.

At first the measurements were performed to confirm the feasibility of the ‘gamma’ condition. Gamma as a function of droplet size is illustrated in Fig. 5, where it remains almost constant for different sizes. The reason for the large standard deviation in the last measurement point is the non-sphericity of water droplets at these large sizes. The minor differences between the gamma values for s- and p- polarized light is attributed to the imperfection of the algorithms for signal peak detection.

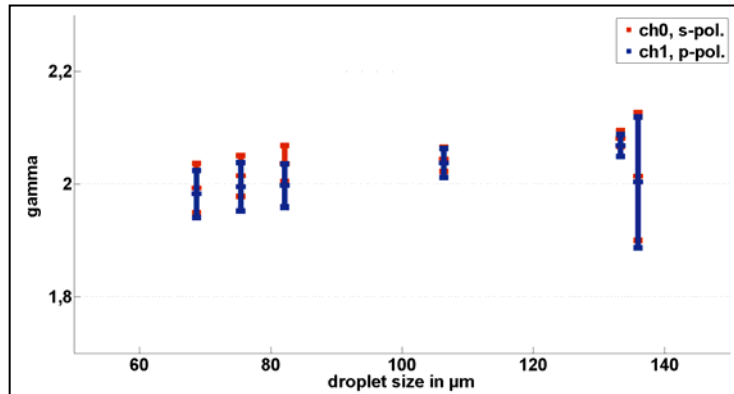


Figure 5: Experimental results for Gamma as a function of droplet size. Refractive index of water approx. 1.34 and the scattering angle was about 160deg.

In further measurements the amplitudes of the time-shift signals for different polarization were analyzed. In Fig. 6 the amplitudes of reflection and second-order refraction are plotted over droplet size for different polarization. As discussed previously these amplitudes are quadratic functions of droplet size. The reflection for s- and p-polarized light are very similar to each other, which can be also verified by Mie-theory.

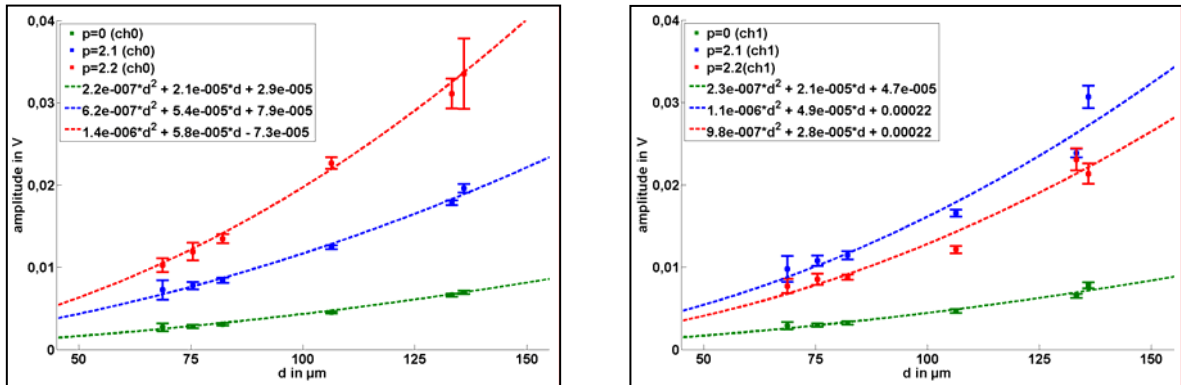


Figure 6: Experimental results for water droplets ($m=1.34$). Amplitudes of time shift signals for s- (left) and p-(right) polarization as a function of droplet size. Lines are curves fitted to the experimental data.

In addition, the velocity of the particle was calculated from the time-shift signal and compared with velocities measured by direct imaging. The result is illustrated in Fig. 7. One can see that the velocity estimation works very well for larger droplets but not for small droplets. The velocities for s- and p-polarization are different from each other. This is caused by the non-optimal signal processing algorithms, because physically they should be the same.

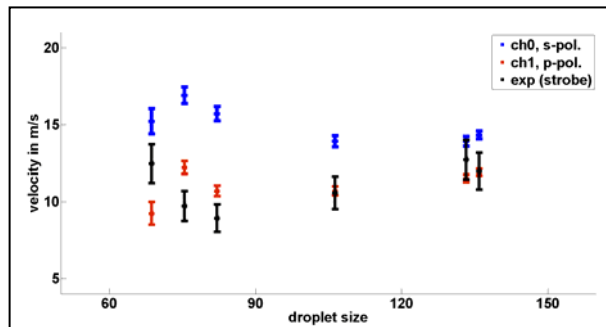


Figure 7: Calculated velocities from time-shift signal for both polarization (ch0 and ch1) and the velocity measured by direct imaging (exp)

Finally the state of polarisation (SOP) of the scattering intensities was measured (Fig. 8). Theoretically SOP should be constantly for different droplet sizes. But some correlation between droplet size and SOP is detected. The possibly reason for the dependency can be the intensity of the surface wave, which was not considered in the SOP calculation but exists in the time-shift signal.

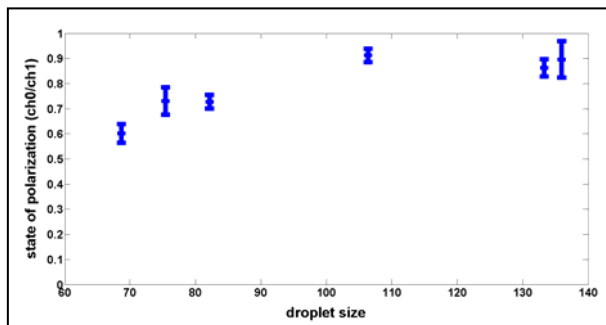


Figure 8: Polarization ratio between time-shift signals.

Discussion of errors and outlook

The accuracy of the measurement method presented here depends strongly on the stability of the measurement volume, which is defined by the laser line and imaging optics of the detector. The imperfection of the algorithms for signal processing also leads to errors, which cannot be avoided alone by invoking the ‘gamma’ condition. For calculation of FWHM of the time-shift signal a Gauss-fit is necessary; however this is computationally a burden. The next problem is the propagation direction of the droplets in the measurement volume, because the droplet can cross the laser line at some distance from beam waist. This will result in a signal width which is broader and will lead to errors in the calculation of the velocity.

These difficulties and refinements of the technique are the topic of on-going development work.

Acknowledgements

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